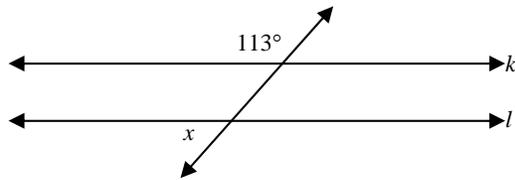# Geometry

Standard 1 - Points, Lines, Angles, and Planes

Students find lengths and midpoints of line segments. They describe and use parallel and perpendicular lines. They find slopes and equations of lines.

G.1.1   Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems.
Example: Find the length and midpoint of the line joining the points A (3, 8) and B (9, 0).

G.1.2   Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.
Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

G.1.3   Understand and use the relationships between special pairs of angles formed by parallel lines and transversals.
Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.

G.1.4   Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.
Example: Find an equation of a line perpendicular to $y = 4x - 2$.

Standard 2 - Polygons

Students identify and describe polygons and measure interior and exterior angles. They use congruence, similarity, symmetry, tessellations, and transformations. They find measures of sides, perimeters, and areas.

G.2.1   Identify and describe convex, concave, and regular polygons.
Example: Draw a regular hexagon. Is it convex or concave? Explain your answer.

G.2.2   Find measures of interior and exterior angles of polygons, justifying the method used.
Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

G.2.3   Use properties of congruent and similar polygons to solve problems.
Example: Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

G.2.4   Apply transformations (slides, flips, turns, expansions, and contractions) to polygons to determine congruence, similarity, symmetry, and tessellations. Know that images formed by slides, flips, and turns are congruent to the original shape.
Example: Use a drawing program to create regular hexagons, regular octagons, and regular pentagons. Under the drawings, describe which of the polygons would be best for tiling a rectangular floor. Explain your reasoning.

G.2.5   Find and use measures of sides, perimeters, and areas of polygons. Relate these measures to each other using formulas.
Example: A rectangle of area 360 square yards is ten times as long as it is wide. Find its length and width.

G.2.6   Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity.
Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6), and (1, 5) a square?

Standard 3 - Quadrilaterals

Students identify and describe simple quadrilaterals. They use congruence and similarity. They find measures of sides, perimeters, and areas.

G.3.1   Describe, classify, and understand relationships among the quadrilaterals square, rectangle, rhombus, parallelogram, trapezoid, and kite.
Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

G.3.2   Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.
Example: Of two similar rectangles, the second has sides three times the length of the first. How many times larger in area is the second rectangle?

G.3.3   Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas.
Example: A section of roof is a trapezoid with length 4 m at the ridge and 6 m at the gutter. The shortest distance from ridge to gutter is 3 m. Construct a model using a drawing program, showing how to find the area of this section of roof.

G.3.4   Use coordinate geometry to prove properties of quadrilaterals, such as regularity, congruence, and similarity.
Example: Is rectangle *ABCD* with vertices at (0, 0), (4, 0), (4, 2), (0, 2) congruent to rectangle *PQRS* with vertices at (-2, -1), (2, -1), (2, 1), (-2, 1)?

Standard 4 - Triangles

Students identify and describe types of triangles. They identify and draw altitudes, medians, and angle bisectors. They use congruence and similarity. They find measures of sides, perimeters, and areas. They apply inequality theorems.

G.4.1   Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular.
Example: Use a drawing program to create examples of right, acute, obtuse, scalene, isosceles, equilateral, and equiangular triangles. Identify and describe the attributes of each triangle.

G.4.2   Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors.
Example: Draw several triangles. Construct their angle bisectors. What do you notice?

G.4.3 Construct triangles congruent to given triangles.
Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

G.4.4 Use properties of congruent and similar triangles to solve problems involving lengths and areas.
Example: Of two similar triangles, the second has sides half the length of the first. The area of the first triangle is 20 cm^2. What is the area of the second?

G.4.5 Prove and apply theorems involving segments divided proportionally.
Example: In triangle *ABC*, $\overline{PQ}$ is parallel to $\overline{BC}$. What is the length of $\overline{AQ}$?
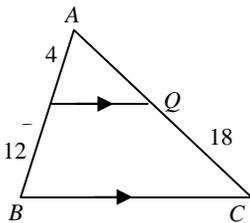
G.4.6 Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles.
Example: In the last example, prove that triangles *ABC* and *APQ* are similar.

G.4.7 Find and use measures of sides, perimeters, and areas of triangles. Relate these measures to each other using formulas.
Example: The gable end of a house is a triangle 20 feet long and 13 feet high. Find its area.

G.4.8 Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem.
Example: Can you draw a triangle with sides of length 7 cm, 4 cm, and 15 cm?

G.4.9 Use coordinate geometry to prove properties of triangles such as regularity, congruence, and similarity.
Example: Draw a triangle with vertices at (1, 3), (2, 5), and (6, 1). Draw another triangle with vertices at (-3, -1), (-2, 1), and (2, -3). Are these triangles the same shape and size?

Standard 5 - Right Triangles

Students prove the Pythagorean Theorem and use it to solve problems. They define and apply the trigonometric relations sine, cosine, and tangent.

G.5.1 Prove and use the Pythagorean Theorem.
Example: On each side of a right triangle, draw a square with that side of the triangle as one side of the square. Find the areas of the three squares. What relationship is there between the areas?

G.5.2 State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.
Example: In triangle *ABC* with right angle at *C*, draw the altitude $\overline{CD}$ from *C* to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

G.5.3 Use special right triangles (30° - 60° and 45° - 45°) to solve problems.
Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

G.5.4   Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles.
Example: In triangle *ABC*, tan *A* = $\frac{1}{5}$. Find sin *A* and cot *A*.

G.5.5   Know and use the relationship $\sin^2 x + \cos^2 x = 1$.
Example: Show that, in a right triangle, $\sin^2 x + \cos^2 x = 1$ is an example of the Pythagorean Theorem.

G.5.6   Solve word problems involving right triangles.
Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

Standard 6 - Circles

Students define ideas related to circles: e.g., radius, tangent. They find measures of angles, lengths, and areas. They prove theorems about circles. They find equations of circles.

G.6.1   Find the center of a given circle. Construct the circle that passes through three given points not on a line.
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

G.6.2   Define and identify relationships among: radius, diameter, arc, measure of an arc, chord, secant, and tangent.
Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

G.6.3   Prove theorems related to circles.
Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

G.6.4   Construct tangents to circles and circumscribe and inscribe circles.
Example: Draw an acute triangle and construct the circumscribed circle.

G.6.5   Define, find, and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).
Example: Find the measure of angle *ABC* in the diagram below.
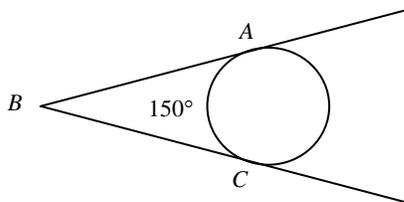
G.6.6   Define and identify congruent and concentric circles.
Example: Are circles with the same center always the same shape? Are they always the same size?

G.6.7   Define, find, and use measures of circumference, arc length, and areas of circles and sectors. Use these measures to solve problems.
Example: Which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer.

G.6.8   Find the equation of a circle in the coordinate plane in terms of its center and radius.
         Example: Find the equation of the circle with radius 10 and center (6, -3).

Standard 7 - Polyhedra and Other Solids

Students describe and make polyhedra and other solids. They describe relationships and symmetries, and use congruence and similarity.

G.7.1   Describe and make regular and nonregular polyhedra.
         Example: Is a cube a regular polyhedron? Explain why or why not.

G.7.2   Describe the polyhedron that can be made from a given net (or pattern). Describe the net for a given polyhedron.
         Example: Make a net for a tetrahedron out of poster board and fold it up to make the tetrahedron.

G.7.3   Describe relationships between the faces, edges, and vertices of polyhedra.
         Example: Count the sides, edges, and corners of a square-based pyramid. How are these numbers related?

G.7.4   Describe symmetries of geometric solids.
         Example: Describe the rotation and reflection symmetries of a square-based pyramid.

G.7.5   Describe sets of points on spheres: chords, tangents, and great circles.
         Example: On Earth, is the equator a great circle?

G.7.6   Identify and know properties of congruent and similar solids.
         Example: Explain how the surface area and volume of similar cylinders are related.

G.7.7   Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.
         Example: An ice cube is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ice cube?

Standard 8 - Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

G.8.1   Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.
         Example: How far does the tip of the minute hand of a clock move in 20 minutes if the tip is 4 inches from the center of the clock?

G.8.2   Decide whether a solution is reasonable in the context of the original situation.
         Example: John says the answer to the problem in the first example is 12 inches. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

G.8.3   Make conjectures about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture.
         Example: Calculate the ratios of side lengths in several different-sized triangles with angles of 90°, 50°, and 40°. What do you notice about the ratios? How might you prove that your observation is true (or show that it is false)?

G.8.4 Write and interpret statements of the form "if – then" and "if and only if."
Example: Decide whether this statement is true: "If today is Sunday, then we have school tomorrow."

G.8.5 State, use, and examine the validity of the converse, inverse, and contrapositive of "if – then" statements.
Example: In the last example, write the converse of the statement.

G.8.6 Identify and give examples of undefined terms, axioms, and theorems, and inductive and deductive proofs.
Example: Do you prove axioms from theorems or theorems from axioms?

G.8.7 Construct logical arguments, judge their validity, and give counterexamples to disprove statements.
Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

G.8.8 Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, and two-column and indirect proofs.
Example: In triangle LMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

G.8.9 Perform basic constructions, describing and justifying the procedures used. Distinguish between constructing and drawing geometric figures.
Example: Construct a line parallel to a given line through a given point not on the line, explaining and justifying each step.